# Assessing disease resistance in chickpeas through the bivariate analysis of Gaussian and binomial traits. 

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## Mixed Models and Experiment Design Lab (MMaEDL) ${ }^{1}$

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NSW Department of Primary Industries | Agriculture

- Motivation
- Context
- Complexities
- The statistics of the problem
- Choice of model
- Results
- Parametric bootstrapping


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- Field experiments are expensive, labour intensive and time consuming
- These factors lead to a complex statistical analysis


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- Both experiments assess a similar set of genetic material (varieties) under artificially induced/amplified PRR pressure
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- Hydroponics experiments provide high throughput low cost alternatives to field experiments


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| Tank | Rack | HRange | HRow | Variety | TotalLeaf | DisLeaf |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 1 | 1 | 1 | A | 6 | 2 |
| 1 | 1 | 2 | 1 | B | 5 | 3 |
| 1 | 1 | 3 | 1 | C |  |  |
| 1 | 1 | 4 | 1 | D | 11 | 6 |
| 1 | 1 | 5 | 1 | F | 13 | 9 |
| 1 | 1 | 6 | 1 | A | 15 | 7 |

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| FCol | FRow | Variety | Risk1 | Dead1 | Risk2 | Dead2 | Risk3 | Dead3 | ExpectedLife |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 1 | A | 16 | 0 | 16 | 12 | 4 | 15 | 1.57 |
| 1 | 2 | B | 16 | 0 | 16 | 0 | 16 | 0 | 9.39 |
| 1 | 3 | C | 16 | 0 | 16 | 0 | 16 | 2 | 4.01 |
| 1 | 4 | D | 10 | 0 | 10 | 7 | 3 | 9 | 1.75 |
| 1 | 5 | E | 19 | 0 | 19 | 2 | 17 | 7 | 2.98 |
| 1 | 6 | A | 16 | 0 | 16 | 4 | 12 | 7 | 2.87 |

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- Reflects the plot structure of each experiment
- Accurately captures the underlying distribution of each measured trait
- Provides a reliable estimate of the genetic correlation between experiments
- We choose a bivariate generalised linear mixed model (GLMM) using ASReml-R


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- To overcome this the two data frames are merged by variety in a non-unique way


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| WPlot | Variety | FCol | FRow | Tank | Rack | HRange | HRow | DL* | TL* | EL* |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | A | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 6 | 1.57 |
| 2 | B | 1 | 2 | 1 | 1 | 2 | 1 | 3 | 5 | 9.39 |
| 3 | C | 1 | 3 | 1 | 1 | 3 | 1 |  |  | 4.01 |
| 4 | D | 1 | 4 | 1 | 1 | 4 | 1 | 6 | 11 | 1.75 |
| 5 | E | 1 | 5 |  |  |  |  |  |  | 2.98 |
| 6 | F |  |  | 1 | 1 | 5 | 1 | 9 | 13 |  |
| 7 | A | 1 | 6 | 1 | 1 | 6 | 1 | 7 | 15 | 2.87 |

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- WPlot arbitrarily joins field and hydroponic observations by common varieties, however as evident in WPlot 5 and 6 not all field and hydroponic observations can be matched


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- $\mathbf{y}_{F}=\left(y_{F_{1}}, \ldots, y_{F_{n_{F}}}\right)^{\top}$ from the field experiment which we assume follows a normal distribution
- We would also like to include random effects $\mathbf{u}=\left(\mathbf{u}_{g}^{\top}, \mathbf{u}_{p_{H}}^{\top}, \mathbf{u}_{\rho_{F}}^{\top}\right)^{\top}$ which are either shared genetic effects $\mathbf{u}_{g}$ or non-shared peripheral effects $\mathbf{u}_{p_{H}}$ and $\mathbf{u}_{p_{F}}$, for the hydroponic and field experiments respectively


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- We can then write the distributions $\mathbf{y}_{H} \mid \mathbf{u}$ and $\mathbf{y}_{F} \mid \mathbf{u}$ conditional on the random effects $\mathbf{u}$ with associated probability density functions (PDF) $f_{Y_{H} \mid U}$ and $f_{Y_{F} \mid U}$


## Model - Hydroponic Conditional Distribution

$$
\begin{gathered}
E\left(y_{H_{i}} \mid \mathbf{u}\right)=\mu_{H_{i}} \\
g\left(\mu_{H_{i}}\right)=\eta_{H_{i}}=\mathbf{x}_{H_{i}}^{\top} \boldsymbol{\tau}+\mathbf{z}_{H_{i}}^{\top} \mathbf{u} \\
V\left(y_{H_{i}} \mid \mathbf{u}\right)=\phi_{H} \mu_{H_{i}}\left(1-\frac{\mu_{H_{i}}}{n_{T L_{i}}}\right)
\end{gathered}
$$

- for $i=1, \ldots, n_{H}$ where $g()$ is the logit link function
- $\mathbf{x}_{H_{i}}$ and $\mathbf{z}_{H_{i}}$ are indicator/covariate vectors of length $c_{X}$ and $c_{Z}$ relating to the fixed $(\boldsymbol{\tau})$ and random ( $\mathbf{u}$ ) effects for the $i$ th observation in the hydroponic experiment
- $\phi_{H}$ represents the dispersion parameter and $n_{T L_{i}}$ are the binomial totals for the $i$ th observation in the hydroponic experiment


## Model - Field Conditional Distribution

$$
\begin{gathered}
E\left(y_{F_{i}} \mid \mathbf{u}\right)=\eta_{F_{i}}=\mathbf{x}_{F_{i}}^{\top} \boldsymbol{\tau}+\mathbf{z}_{F_{i}}^{\top} \mathbf{u} \\
V\left(y_{F_{i}} \mid \mathbf{u}\right)=\sigma_{F}^{2}
\end{gathered}
$$

- for $i=1, \ldots, n_{F}$
- $\mathbf{x}_{F_{i}}$ and $\mathbf{z}_{F_{i}}$ are indicator/covariate vectors of length $c_{X}$ and $c_{Z}$ relating to the fixed $(\boldsymbol{\tau})$ and random $(\mathbf{u})$ effects for the $i$ th observation in the field experiment
- $\sigma_{F}^{2}$ is the field residual variance


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$$
\left[\begin{array}{l}
\mathbf{u}_{g} \\
\mathbf{u}_{p_{H}} \\
\mathbf{u}_{p_{F}}
\end{array}\right] \sim N\left(\left[\begin{array}{l}
\mathbf{0} \\
\mathbf{0} \\
\mathbf{0}
\end{array}\right],\left[\begin{array}{ccc}
\mathbf{G}_{g} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{G}_{p_{H}} & \mathbf{0} \\
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\end{array}\right]\right)
$$

- Or more generally $\mathbf{u} \sim N(\mathbf{0}, \mathbf{G})$ with associated PDF $f_{U}$
- For convenience we allow $\boldsymbol{\kappa}$ to contain all parameters in $\mathbf{G}_{g}, \mathbf{G}_{p_{H}}$ and $\mathbf{G}_{p_{F}}$ along with $\phi_{H}$ and $\sigma_{F}^{2}$


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& =\log \left(\int f_{Y_{H} \mid U} f_{U} d \mathbf{u}\right)+\log \left(\int f_{Y_{F} \mid U} f_{U} d \mathbf{u}\right)
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\end{aligned}
$$

- The first term of which is not analytically tractable and hence requires an alternative to classical likelihood inference - therefore PQL is used to approximate the likelihood (Collins, 2008)


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$$

- Here $\mathbf{G}_{g_{a}}=\boldsymbol{\Sigma}_{a} \otimes \mathbf{K}$ and $\mathbf{G}_{g_{e}}=\boldsymbol{\Sigma}_{e} \otimes \mathbf{I}_{c_{g}}$ where $\mathbf{K}$ is a $\left(c_{g} \times c_{g}\right)$ known genomic relationship matrix (GRM) formed via marker scores


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- Where $\boldsymbol{\Sigma}_{a}=\left[\begin{array}{cc}\sigma_{a_{H}} & \sigma_{\text {aHF }} \\ \sigma_{a_{\text {aF }}} & \sigma_{a_{\text {aF }}}\end{array}\right]$ and $\boldsymbol{\Sigma}_{e}=\left[\begin{array}{cc}\sigma_{e_{H}} & \sigma_{e_{\text {eF }}} \\ \sigma_{e_{\text {HF }}} & \sigma_{e_{F}}\end{array}\right]$ is known as an unstructured parameterisation


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- In the case of the non-genetic effects, we specify $\mathbf{G}_{p_{H}}$ and $\mathbf{G}_{p_{F}}$ as block diagonal matrices


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\boldsymbol{\Sigma}_{a}=\left[\begin{array}{cc}
\lambda_{a_{H}}^{2} & \lambda_{a_{H}} \lambda_{a_{F}} \\
\lambda_{a_{H}} \lambda_{a_{F}} & \lambda_{a_{F}}^{2}+\psi_{a_{F}}
\end{array}\right] \text { and } \boldsymbol{\Sigma}_{e}=\left[\begin{array}{cc}
\lambda_{e_{H}}^{2} & \lambda_{e_{H}} \lambda_{e_{F}} \\
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\lambda_{e_{H}} \lambda_{e_{F}} & \lambda_{e_{F}}^{2}+\psi_{e_{F}}
\end{array}\right]
$$

- This parameterisation is known as "reduced rank + diag" and is convenient as estimates of the $\lambda_{i j}, i=a, e, j=H, F$ cannot go outside the parameter space unlike the " $\sigma$ " parameters in the unstructured parameterisation


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- Due to the non-Gaussian nature of the response recorded in the hydroponic experiment we use PQL, an approximate likelihood method as the likelihood analytically intractable
- For the genetic effects, which are our main interest, we specify a variance structure which allows for covariance between experiments


## Model - ASReml

```
asreml(fixed = cbind(DL,EL) ~ trait + trait:VarietyDrop,
    random =~ rr(trait):vm(Variety, K) + at(trait,"EL"):vm(Variety, K) +
        rr(trait):ide(Variety) + at(trait,"EL"):ide(Variety) +
        at(trait, "DL"):Tank + at(trait, "DL"):Tank:Rack +
        at(trait, "DL"):Tank:Rack:HRow + at(trait, "DL"):Tank:Rack:HRange +
        at(trait, "EL"):FRow + at(trait, "EL"):FCol,
    residual = ~id(WPlot):diag(trait),
    data = df,
    family = list(asr_binomial(total = "TL"), asr_gaussian()),
    na.action = na.method(x="include",y="include"))
```


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- The researcher would also like a confidence interval on this estimate
- To do so while also assessing potential bias in the PQL estimates, a known issue impacting parameter estimation (Breslow \& Lin, 1995) we implement parametric bootstrapping


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- $i=1, \ldots, p$ where $p$ is the number of parameters, $\hat{\theta}_{i}^{(j)}$ is the estimate of the $i$ th parameter at the $j$ th iteration with $t$ being the final iteration
- After 50 iterations simulations where $\Delta_{\text {norm }}>0.001$ were excluded


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## Parametric Bootstrapping - Sampling Distribution

| Parameter | Obs | BootMean |
| :---: | :---: | :---: |
| $\sigma_{\text {Tank }}^{2}$ | 0.082 | 0.077 |
| $\sigma_{\text {Rack }}^{2}$ | 0.022 | 0.022 |
| $\sigma_{\text {FRow }}^{2}$ | 0.000 | 0.014 |
| $\sigma_{\text {FCol }}^{2}$ | 0.013 | 0.025 |
| $\sigma_{\text {HRange }}^{2}$ | 0.038 | 0.035 |
| $\sigma_{\text {HRow }}^{2}$ | 0.092 | 0.088 |
| $\psi_{a_{F}}$ | 0.000 | 0.023 |
| $\psi_{e_{F}}$ | 2.458 | 1.617 |
| $\lambda_{a_{H}}$ | 0.151 | 0.335 |
| $\lambda_{a_{F}}$ | -1.074 | -0.032 |
| $\lambda_{e_{H}}$ | 0.376 | 0.997 |
| $\lambda_{e_{F}}$ | -0.591 | -0.874 |
| $\phi_{H}$ | 1.095 | 0.965 |
| $\sigma_{F}^{2}$ | 3.593 | 3.571 |

$\sigma_{j}^{2}, j=$ Tank, Rack, FRow, FCol, HRange and HRow are the variance components associated with the peripheral random effects, $\psi_{a_{F}}, \psi_{e_{F}}, \lambda_{a_{H}}$,
$\lambda_{a_{F}}, \lambda_{e_{H}}$ and $\lambda_{e_{F}}$ are the genetic parameters and $\phi_{H}$ and $\sigma_{F}^{2}$ are the hydroponic dispersion and field residual variance parameter respectively

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- The resulting analysis gave a quantification of the level of agreement between PRR resistance for the two experiments
- Parametric bootstrapping provided assessment of parameter estimation bias and indicated the associated $95 \%$ bootstrap Cl for the total genetic correlation does not contain 0


## Paper

## Rapid and high throughput hydroponics phenotyping method for evaluating chickpea

 resistance to Phytophthora root rotMuhammad A. Asif ${ }^{1 *}$, Sean L. Bithell ${ }^{2}$, Ramethaa Pirathiban ${ }^{3}$, Brian R. Cullis ${ }^{3}$, David Hughes ${ }^{3}$, Aidan McGarty ${ }^{3}$, Nicole Dron ${ }^{2}$ and Kristy Hobson ${ }^{1}$
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${ }^{2}$ New South Wales Department of Primary Industries, Tamworth Agricultural Institute, Tamworth, New South Wales, Australia
${ }^{3}$ Centre for Biometrics and Data Science for Sustainable Primary Industries, National Institute for Applied Statistics and Research Australia, School of Mathematics and Applied Statistics, University of Wollongong, Australia >
*Corresponding author: ahsan.asif@dpie.nsw.gov.au

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